Cognitive Dynamic Systems

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1. What is Cognition?

According to the Oxford English Dictionary:

Cognition is

Knowing,	**Knowledge** **Knowledge Acquisition** **Knowledge Representation** **Contextual Knowledge** **Storage of Knowledge**
Perceiving,	**Perception** **Sensing of the Environment** **Adaptation to the Environment** **Learning from the Environment**
or Conceiving	**Dealing with Uncertainty:** **1. Probabilistic Reasoning** **2. Hypothesizing and Decision-making** **“The Bayesian framework”**
an Act	**Control** **Approximate Dynamic Programming**
etc.	**Energy Efficiency** **Robustness**

The human brain has all these attributes, and there is plausible evidence for the Bayesian framework -- hence the “Bayesian brain”.

2. Cognitive Dynamic Systems Defined

A Cognitive Dynamic System is a system that processes information over the course of time by performing the following functions:

- ***Sense*** **the environment;**
- ***learn*** **from the environment and adapt to its statistical variations;**
- **build a** ***predictive model*** **of prescribed aspects of the environment**

and thereby develop ***rules of behaviour*** **for the execution of prescribed tasks, in the face of environmental uncertainties,** ***efficiently and reliably in a cost-effective manner.***

3. Emerging Applications

Cognitive radio
(Candidate for 5th generation wireless communications)

Cognitive radar

Cognitive car

Cognitive genome

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Cognitive optimization

Cognitive software

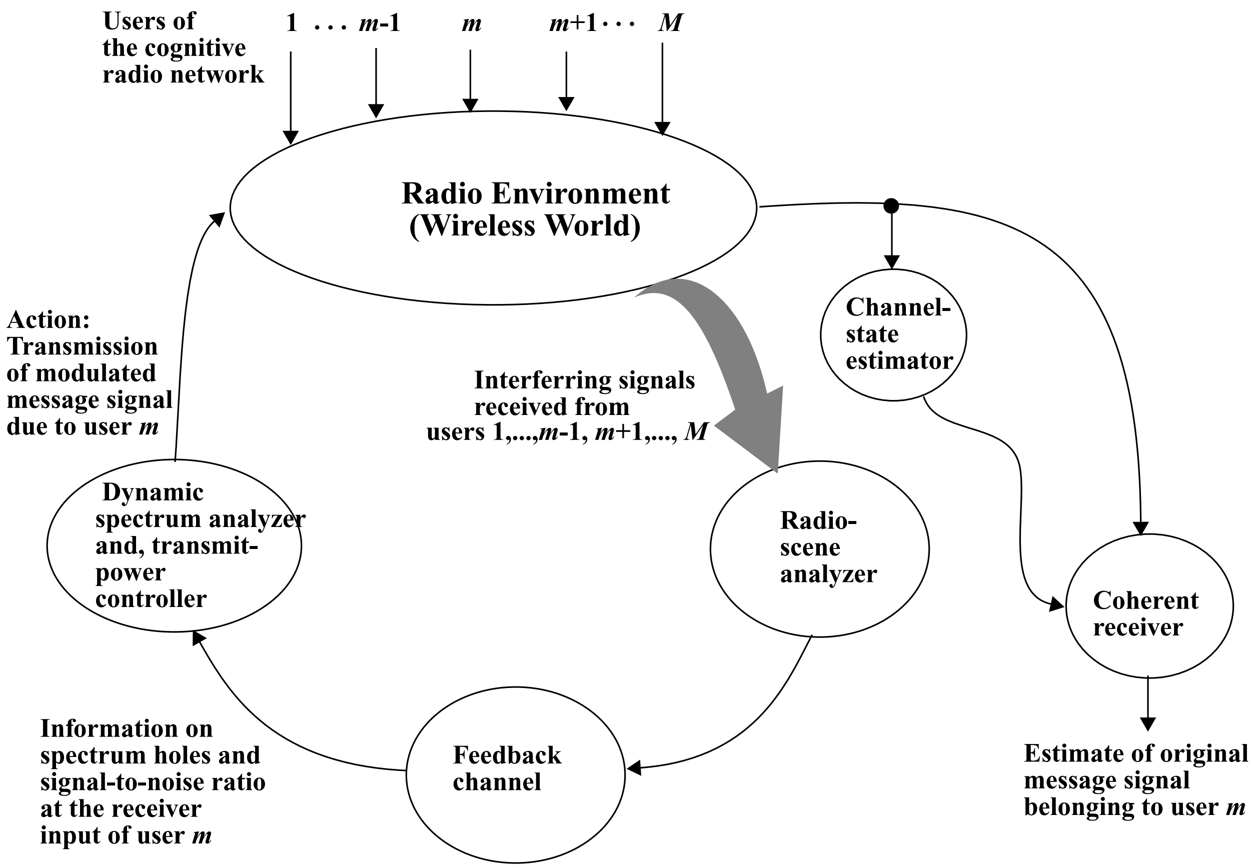
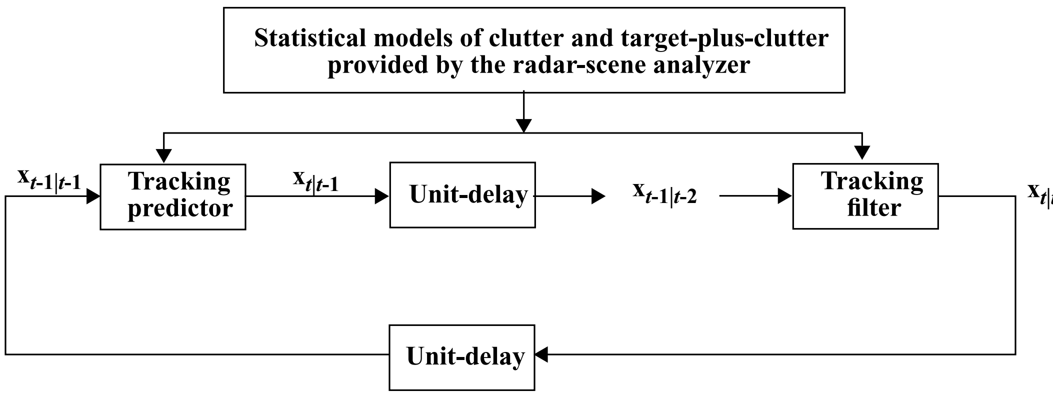


Figure 1: Cognitive signal-processing cycle for user *m* of cognitive radio network; the diagram also includes elements of the receiver of user *m*



Notations

***t*: discrete-time**

$x_{t|t}$: filtered state vector of probabilities of targets being present in the search space at *t* given spectral measurements up to and including time *t*

The other data vectors in the diagram are similarly defined

Figure 2: Block diagram of the Bayesian direct filtering system

4. Global Feedback

"A Facilitator of Computational Intelligence"

- **The human brain is a living example of a cognitive dynamic system with global feedback in many of its parts, be that the visual system, auditory system, or motor control.**

- **Global feedback is responsible for the coordination of different constituents of a cognitive dynamic system.**

- **Global feedback is an inherent property of all cognitive dynamic systems, but global feedback by itself will *not* make a dynamic system cognitive.**

5. Why sub-optimality should be the objective of cognitive dynamic systems?

- **Optimality of performance versus robustness of behaviour.**

- **Global optimality of a cognitive dynamic system is not practically feasible:**

  - **Infeasible computability**
  - **Curse-of-dimensionality**
  - **Large-scale nature of the system**

  Hence, the practical requirement of having to settle for a sub-optimal solution of the system design

- **Trade-off global optimality for computational tractability and robust behaviour.**

Criterion for sub-optimality

DO AS BEST AS YOU CAN, AND NOT MORE

- **This statement is the essence of what the human brain does on a daily basis:**

  Provide the "best" solution in the most reliable fashion for the task at hand, given limited resources.

- **Key question: How do we define "best"?**

6. The Bayesian Filter: A powerful tool for cognitive information processing

Problem statement:

> **Given a nonlinear dynamic system, estimate the *hidden state* of the system in a recursive manner by processing a sequence of *noisy observations* dependent on the state.**

- **The Bayesian filter provides a unifying framework for the optimal solution of this problem, at least in a conceptual sense.**
- **Unfortunately, except in a few special cases, the Bayesian filter is not implementable in practice -- hence the need for approximation.**

Bayesian Filter (continued)

State-space Model

1. **System (state) Model**

$$\mathbf{x}_{t+1} = \mathbf{a}(\mathbf{x}_t) + \omega_t$$

2. **Measurement model**

$$\mathbf{y}_t = \mathbf{b}(\mathbf{x}_t) + \nu_t$$

where t = discrete time
$\mathbf{x}_t$ = state at time *t*
$\mathbf{y}_t$ = observation at time
ω_t = dynamic noise
ν_t = measurement noise

Assumptions:

- **Nonlinear functions a(.) and b(.) are known**
- **Dynamic noise ω_t and measurement noise ν_t are statistically independent Gaussian processes of zero mean and known covariance matrices.**

Bayesian filter (continued)

Time-update equation:

$$\underbrace{p(\mathbf{x}_t|\mathbf{Y}_{t-1})}_{\text{predictive distribution}} = \int_{R^n} \underbrace{p(\mathbf{x}_t|\mathbf{x}_{t-1})}_{\text{prior distribution}} \underbrace{p(\mathbf{x}_{t-1}|\mathbf{Y}_{t-1})}_{\text{old posterior distribution}} d\mathbf{x}_{t-1}$$

where R^n denotes the n-dimensional state space.

Measurement-update equation:

$$\underbrace{p(\mathbf{x}_t|\mathbf{Y}_t)}_{\text{Updated posterior distribution}} = \frac{1}{Z_t} \underbrace{p(\mathbf{x}_t|\mathbf{Y}_{t-1})}_{\text{Predictive distribution}} \underbrace{l(\mathbf{y}_t|\mathbf{x}_t)}_{\text{Likelihood function}}$$

where Z_t is the normalizing constant defined by

$$Z_t = \int_{R^n} p(\mathbf{x}_t|\mathbf{Y}_{t-1}) l(\mathbf{y}_t|\mathbf{x}_t) d\mathbf{x}_t$$

Bayesian Filter (continued)

- **The celebrated Kalman filter is a special case of the Bayesian filter, assuming that the dynamic system is linear.**

- **Except for this special case and couple of other cases, exact computation of the predictive distribution $p(\mathbf{x}_t | \mathbf{Y}_{t-1})$ is not feasible.**

- **We therefore have to abandon optimality and be content with a sub-optimal nonlinear filtering algorithm that is computationally tractable.**

Bayesian Filtering (continued)

Two Approaches for Approximate Nonlinear Filtering

1. **Direct numerical approximation of the posterior in a local sense:**
   - **Extended Kalman filter**
   - **Unscented Kalman filter (Julier, Ulhmann and Durrant-Whyte, 2000)**
   - **Central-difference Kalman filter (Nörgaard, Poulson, and Ravn, 2000).**
   - **Cubature Kalman filter (Arasaratnam and Haykin, 2008).**

2. **Indirect numerical approximation of the posterior in a global sense:**
   - **Particle filters (Gordon, Salmond, and Smith, 1993)**
   - **Roots embedded in Monte Carlo simulation**
   - **Computationally demanding**

Extended Kalman Filter

- **Linearize the system model around the filtered estimate $\hat{\mathbf{x}}_{t|t}$, and linearize the measurement model around the predicted estimate $\hat{\mathbf{x}}_{t|t-1}$**

- **Attributes and Limitations**

  (i) The EKF is simple to implement

  (ii) Estimation accuracy of the EKF is good for nonlinearities of a mild sort; otherwise, it is often highly suboptimal.

7. The Cubature Kalman Filter

(Arasaratnam and Haykin, IEEE Trans. Automatic Control, accepted for publication subject to revisions)

- **At the heart of the Bayesian filter, we have to compute integrals whose integrand is expressed in the form**

(Nonlinear function) x **(Gaussian function)**

- **The challenge is to numerically approximate the integral so as to completely preserve second-order information about the state x that is contained in the sequence of observations**

- **The computational tool that accommodates this requirement is the *cubature rule* (Cools, 1997).**

Cubature Kalman Filter (continued)

The Cubature Rule

- **In mathematical terms, we have to compute an integral for the generic form**

$$h(\mathbf{f}) = \int_{R^n} \underbrace{\mathbf{f}(\mathrm{x})}_{\text{Arbitray nonlinear function}} \underbrace{\exp\left(-\frac{1}{2}\mathrm{x}^T \mathrm{x}\right)}_{\text{Normalized Gaussian function of zero mean and unit covariance function}} d\mathrm{x} \qquad \textbf{(1)}$$

- **To do the computation, the key step is to make a change of variables from the Cartesian coordinate system (in which the vector x is defined) to a spherical-radial coordinate system:**

$\mathbf{x} = r\mathbf{z}$ subject to $\mathbf{z}^T\mathbf{z} = 1$ and $\mathbf{x}^T\mathbf{x} = r^2$

where $0 \leq r < \infty$

Cubature Kalman Filter (continued)

- **We may thus express *I* as the radial integral**

$$I = \int_0^\infty S(r) r^{n-1} \exp(-r^2) dr$$

where *S*(*r*) is defined by the spherical integral

$$S(r) = \int_{U_n} f(r\mathbf{z}) d\sigma(\mathbf{z})$$

where σ(.) is the spherical surface measure on the region

$$U_n = \{\mathbf{z}, \text{subject to } \mathbf{z}^T\mathbf{z} = 1\}$$

- **Working through a fair amount of mathematical details, we finally arrive at the desired *linear approximation*:**

$$h(f) = \int_{R^n} \mathbf{f}(\mathbf{x}) \underbrace{N(\mathbf{x};\mathbf{0}, \mathbf{I})}_{\text{Standard Gaussian function}} d\mathbf{x}$$

$$\approx \sum_{i=1}^{2n} \omega_i \mathbf{f}\{\xi_i\}$$

where $\{\xi_i\}$ = cubature representations of the state vector x.

$$\omega_i = \frac{1}{m}, \quad i = 1, 2, ..., m = 2n$$

- **The set $\{\xi_i, \omega_i\}_{i=1}^{2n}$ constitutes the cubature points used to numerically compute integrals of the form defined in Eq. (1).**

Parameter Updates of the Cubature Kalman Filter

Time update

$$\hat{\mathbf{x}}_{n|n-1} = \mathbf{E}[\mathbf{x}_n | \mathbf{Y}_{n-1}]$$

$$= \int_{R^M} \underbrace{\mathbf{a}(x_{n-1})}_{\text{Nonlinear state function}} \underbrace{N(\mathbf{x}_{n-1};\hat{\mathbf{x}}_{n-1|n-1}, \mathbf{P}_{n-1|n-1})}_{\text{Gaussian distribution}} d\mathbf{x}_{n-1}$$

Measurement update

$$N\left(\underbrace{\begin{bmatrix}\mathbf{x}_n \\ \mathbf{y}_n\end{bmatrix}}_{\text{Joint variables}};\underbrace{\begin{bmatrix}\hat{\mathbf{x}}_{n|n-1} \\ \hat{\mathbf{y}}_{n|n-1}\end{bmatrix}}_{\text{Joint mean}},\underbrace{\begin{bmatrix}\mathbf{P}_{n|n-1} & \mathbf{P}_{xy,\,n|n-1} \\ \mathbf{P}_{yx,\,n|n-1} & \mathbf{P}_{yy,\,n|n-1}\end{bmatrix}}_{\text{Joint covariance matrix}}\right)$$

$$\hat{\mathbf{y}}_{n|n-1} = \int_{R^M} \underbrace{\mathbf{b}(\mathbf{x}_n)}_{\text{Nonlinear measurement function}} \underbrace{N(\mathbf{x}_n;\hat{\mathbf{x}}_{n|n-1}, \mathbf{P}_{n|n-1})}_{\text{Gaussian function}} d\mathbf{x}_n$$

$$P_{yy,\,n|n-1} = \int_{R^M} \underbrace{\mathbf{b}(\mathbf{x}_n)\mathbf{b}^T(\mathbf{x}_n)}_{\text{Outer product of nonlinear measurement function with itself}} \underbrace{N(\mathbf{x}_n;\hat{\mathbf{x}}_{n|n-1}, \mathbf{P}_{n|n-1})}_{\text{Gaussian function}} d\mathbf{x}_n - \underbrace{\hat{\mathbf{y}}_{n|n-1}\hat{\mathbf{y}}^T_{n|n-1}}_{\text{Outer product of the estimate } \hat{\mathbf{y}}_{n|n-1} \text{ with itself}} + \underbrace{\mathbf{Q}_{\nu,\,n}}_{\text{Covariance matrix measurement noise}}$$

$$\mathbf{P}_{xy,\,n|n-1} = \mathbf{P}_{yx,\,n|n-1}$$

$$= \int_{R^M} \underbrace{\mathbf{x}_n\mathbf{b}^T(\mathbf{x}_n)}_{\text{Outer product of } \mathbf{x}_n \text{ with } \mathbf{b}(\mathbf{x}_n)} \underbrace{N(\mathbf{x}_n;\hat{\mathbf{x}}_{n|n-1}, \mathbf{P}_{n|n-1})}_{\text{Gaussian function}} d\mathbf{x}_n - \underbrace{\hat{\mathbf{x}}_{n|n-1}\hat{\mathbf{y}}^T_{n|n-1}}_{\text{Outer product of the estimates } \hat{\mathbf{x}}_{n|n-1} \text{ and } \hat{\mathbf{y}}_{n|n-1}}$$

Recursive Cycle of the Cubature Kalman Filter

- **The Kalman gain is computed as**

  $$\mathbf{G}_n = \mathbf{P}_{xy,\, n|n\text{-}1}\mathbf{P}^{-1}_{yy,n|n\text{-}1}$$

  where $\mathbf{P}^{-1}_{yy,n|n\text{-}1}$ **is the inverse of the covariance matrix** $\mathbf{P}^{-1}_{yy,n|n\text{-}1}$.

- **Upon receiving the new observation** $\mathbf{y}_n$**, the filtered estimate of the state** $\mathbf{x}_n$ **is computed in accordance with the predictor-corrector formula:**

  $$\underbrace{\hat{\mathbf{x}}_{n|n}}_{\substack{\text{Updated}\\\text{estimate}}} = \underbrace{\hat{\mathbf{x}}_{n|n\text{-}1}}_{\substack{\text{Old}\\\text{estimate}}} + \underbrace{\mathbf{G}_n}_{\substack{\text{Kalman}\\\text{gain}}}(\underbrace{\mathbf{y}_n - \hat{\mathbf{y}}_{n|n\text{-}1}}_{\text{Innovations process}})$$

- **Correspondingly, the covariance matrix of the filtered state estimation error is computed as shown by**

  $$\mathbf{P}_{n|n} = \mathbf{P}_{n|n\text{-}1} - \mathbf{G}_n\mathbf{P}_{yy,\, n|n\text{-}1}\mathbf{G}_n^T$$

Updated posterior distribution

$$p(\mathbf{x}_n|\mathbf{Y}_n) = N(\mathbf{x}_n;\hat{\mathbf{x}}_{n|n}, \mathbf{P}_{n|n})$$

Properties of the Cubature Kalman Filter

Property* 1*:* The cubature Kalman filter (CKF) is a *derivative-free on-line sequential-state estimator.

***Property* 2: Approximations of the moment integrals are all *linear* in the number of function evaluations.**

***Property* 3: Computational complexity of the cubature Kalman filter grows as n^3, where n is the dimensionality of the state space.**

***Property* 4: The cubature Kalman filter *completely preserves second-order information* about the state that is contained in the observations.**

***Property* 5: The cubature Kalman filter *inherits properties of the linear Kalman filter*, including square-root filtering for improved accuracy and reliability.**

***Property* 6: The cubature Kalman filter is the closest known direct approximation to the Bayesian filter, outperforming the extended Kalman filter and the central-difference Kalman filter:**

It eases the curse-of-dimensionality problem

but, by itself, does not overcome it.

Computer Experiment: Pattern Classification

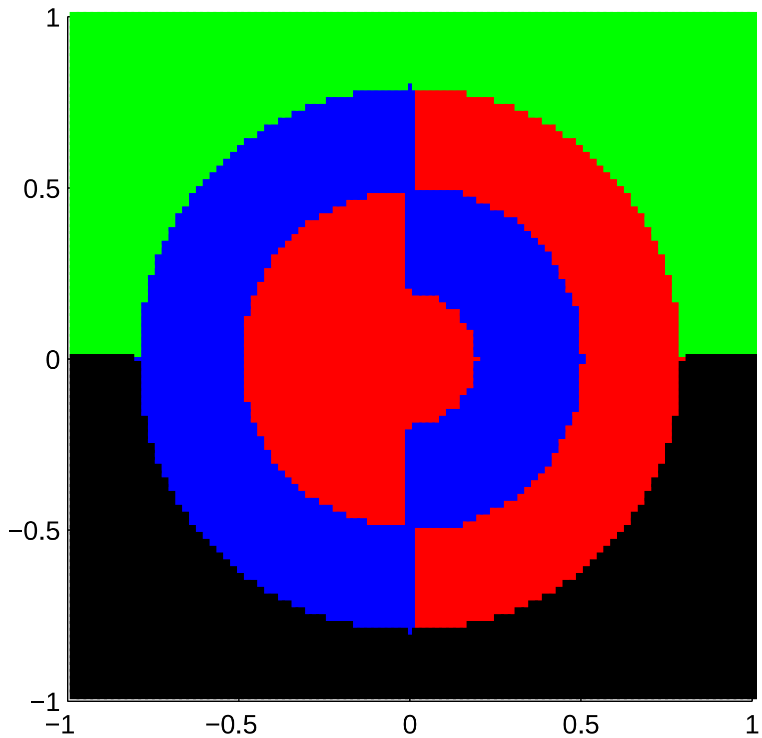


Figure 3: True classification regions

Supervised Learning

Training sample: $\{\mathbf{u}_t, \mathbf{d}_t\}$

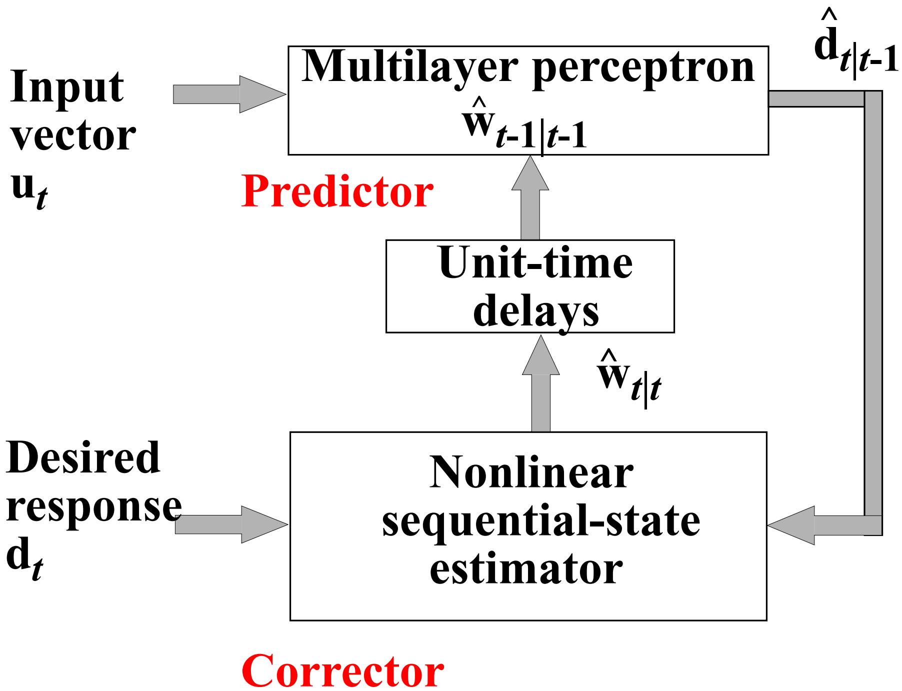


Figure 4: Block diagram of supervised learning machinery

Experimental Setup

- **Used a 2-5-5-4 FFNN with softmax output nonlinearity and the mean squared-error criterion.**

  Total number of adjustable weights: 55 plus biases

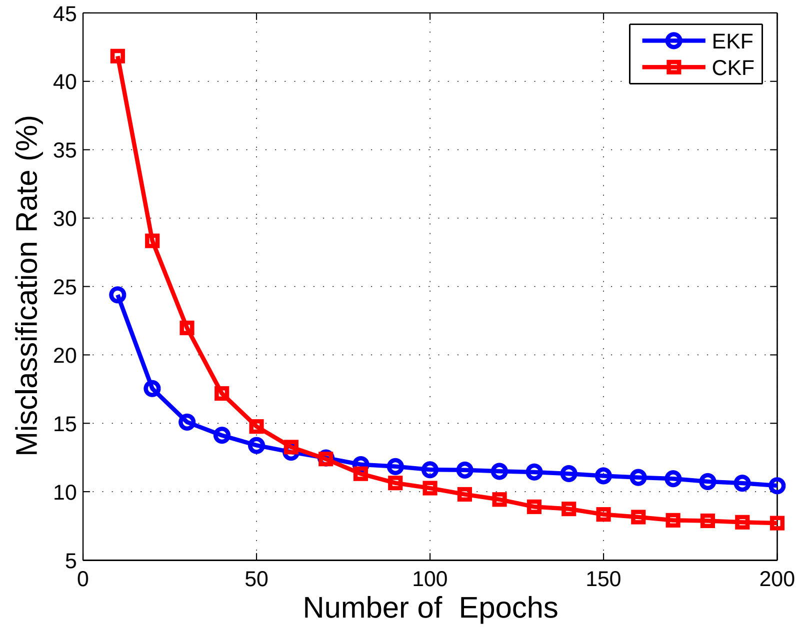
- **1000 training examples drawn randomly from the square region.**

- **DSSM:**
  Process equation: $w_t = w_{t\text{-}1} + \omega_t$
  Measurement equation: $y_t = b(w_t, x_t) + \nu_t$

- **Two training algorithms: EKF, and square-root Kalman filter (SCKF).**

- **To check robustness of filters, 10% of the training examples were mislabeled.**

Performance Comparison



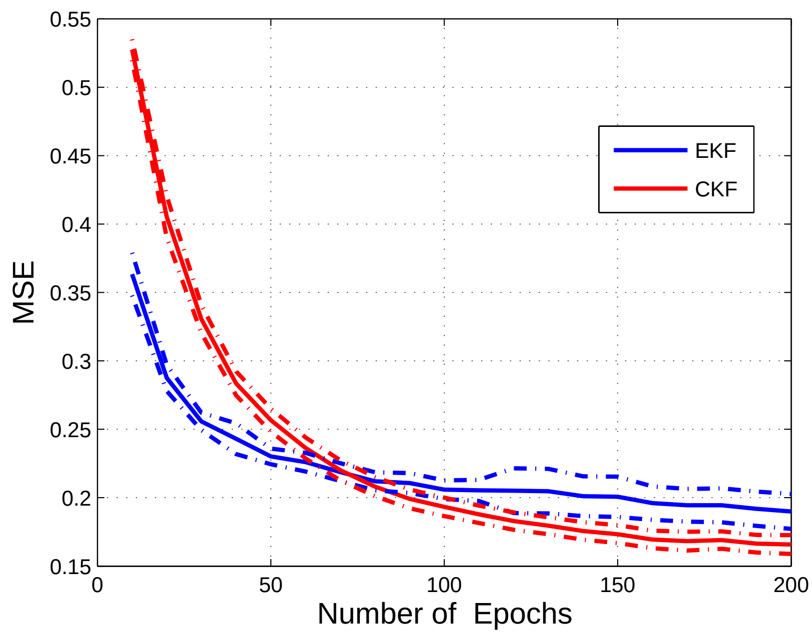


Figure 5:

9. On-going Research Projects in My Laboratory

(i) **Cubature Kalman Filter:**

- Large-scale system applications involving pattern recognition and approximate dynamic programming.

(ii) **Cognitive Radio Networks:**

- Spectrum sensing
- Robust transmit power control
- Dynamic spectrum management
- Emergent behaviour

(iii) **Cognitive Radar Networks:**

- Sub-optimal control of inexpensive (surveillance) radar sensors, given limited computational resources

(iv) **Cocktail Party Processor:**

- Computational auditory scene analysis

10. Concluding Remark

“Cognitive Dynamic Systems”

are

A Way of the Future

in

The 21st Century

Acknowledgements

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The new Website

http://soma.mcmaster.ca

Cognitive Dynamic Systems Workshop, Niagara-on-the-Lake, May 2008, is available and slides can be downloaded from the following link

http://soma.mcmaster.ca/cds2008.php